GYRO-FREQUENCY IN THE IONOSPHERIC REGIONS

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ABSTRACT. Gyro-frequencies in the E, F1 and F2 regions of the ionosphere over Haringhata (Geographic lat. 22.9°N, Geomag. lat. 12.5°N) have been calculated from the measurements of ordinary (f_0) and extraordinary (f_x) critical frequencies. It is found that the magnetic fields calculated therefrom are higher than those expected from extrapolation of the magnetic field at ground-level to the heights of the regions by inverse cube law. Further, the E region gyro-frequency has a marked semi-diurnal variation with a minimum near midday. This result is similar to that obtained by Scott at high latitudes. No diurnal variation is found in the F1 and F2 regions, though, the values near midday in the F1 region are lower. Possible causes of the E-region gyro-frequency variation are discussed. No satisfactory explanation is, however, obtainable.

In the F2 region, the average winter value of the gyro-frequency is found to be about 9% greater than the summer value. Seasonal variation, though of a larger magnitude (20%), has also been obtained by Scott at high latitudes.

The frequency difference (f_x—f_0) is found to be dependent on the ordinary critical frequency (f_0), the values at high frequency being lower than those at lower frequency. This is as may be expected for the case of quasi-transverse propagation.

INTRODUCTION

Appleton and Builder (1933) were the first to show how the difference of the critical frequencies (f_x—f_0) is related to the intensity of the magnetic field in the ionosphere and hence to the so-called gyro-frequency (f_H). Many estimations of the magnetic field and of the gyro-frequency have been made in different parts of the world from the measured values of f_x—f_0, both for the E and the F region. However, as the brief accounts of the experiments given below show there are many unexpected anomalous results. These have not all been fully explained.

Gryo-frequency in the F region at Slough was determined by Appleton by the above method in 1934. The value of the magnetic field in the F region was found to be 0.42 Gauss. This was consistent with that obtained from extrapolation by inverse cube law of the ground level field. Scott in 1950 estimated the magnetic field in the F region at arctic stations by the same method. The results, however, showed that the calculated field from the gyro-frequency is higher than the extrapolated value. Scott also found large diurnal, seasonal and other irregular variations in the calculated field. The field obtained from the longitudinal mode was usually found to be lower than that from the transverse mode. These phenomena are ascribed by Scott as due to the ray path deflections (figure 2) coupled with normal latitude gradients of ionization.

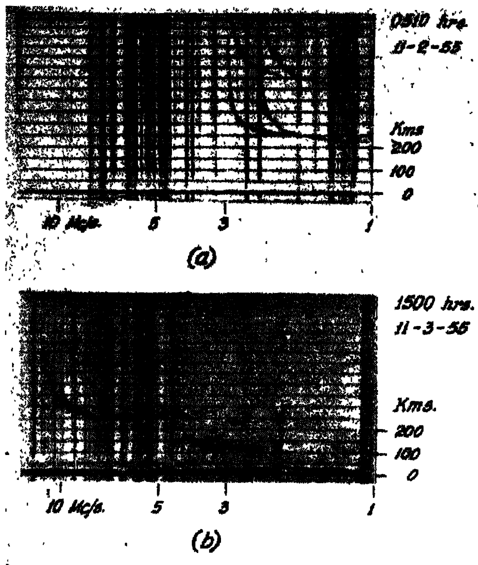

Fig. 1. Typical $h'-f$ records obtained at Haringhata.

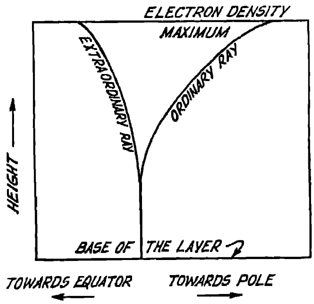

Fig. 2. Diagram illustrating the deviations of the ordinary and extraordinary rays in the Ionosphere.

Scott (1951) has also estimated the magnetic field in the E regions at high latitude arctic stations by the same method. His results show that the calculated magnetic field was lower than the extrapolated value. At one of the stations, a large semi-diurnal variation of the gyro-frequency was found. Scott has suggested that this may be due to a variable concentration of heavy ions rising over 4,000 times the density of free electrons.

THEORETICAL CONSIDERATION

In the case of free-electron ionosphere the gryo-frequency f_H is given by

$$f_H = \frac{H \cdot e_e}{2\pi m_e} \qquad \dots (1)$$

where H = magnetic field in the ionosphere

e_e = charge of an electron

and m_e = mass of an electron.

It can be shown that in such an ionosphere when critical frequency is greater than the gyro-frequency and the mode of propagation for the frequency is quasi-transverse.

$$f_H = \frac{f_x^2 - f_0^2}{f_x} = 2\Delta f - \frac{\Delta^2 f}{f_x} \qquad \dots (2)$$

where $\Delta f = f_x - f_0$

From above, the value of H in terms of f_x and f_0 is given by

$$H = \frac{2\pi m_e}{e_e} \left\{ 2\Delta f - \frac{\Delta^2 f}{f_x} \right\} \qquad \dots (3)$$

We can make an independent estimation of H at the height (h) of the ionospheric regions from the intensity of the magnetic field (H_0) at the surface of the earth by the inverse cube law:

$$H = H_0 \left(\frac{R}{R+h} \right)^3 \qquad \dots (4)$$

where R is the radius of the earth.

Eq. (2) is for the case of free electron ionosphere i.e. when heavy ions are absent. In the presence of heavy-ions, Scott (1951) has shown that

$$\frac{\lambda}{\mu} = \frac{f_x\,[f_H \cdot f_x - (f_x^2 - f_0^2)]}{(f_x - f_H)(f_x^2 - f_0^2)} \qquad \dots (5)$$

where $\lambda = \frac{e_i}{e_e} \cdot \frac{N_i}{N_e}$

$$\mu = \frac{e_e \cdot m_i}{e_i m_e}$$

e_i = charge of ion

m_i = mass of ion

N_i = density of ion

and N_e = density of electron

The above equation may be utilised to find the values of λ from the measured values of f_x and f_o for assumed values of μ and f_H. For this purpose, the value of f_H obtained by extrapolation of the groun level magnetic field by inverse cube law may be substituted.

Eq. (2), which is valid for quasi-transverse propagation can also be written as

$$f^2_x - f_H f_x - f^2_0 = 0 \qquad \dots \quad (6)$$

When solved for f_x, we get

$$f_x = \tfrac{1}{2}\left\{ f_H + 2f_0 \sqrt{1 + \left(\frac{f_H}{2f_0}\right)^2}\right\} \qquad \dots \quad (7)$$

If $f_H < 2f_0$, we get from Eq. (7)

$$\Delta f = f_x - f_0 = \frac{f_H}{2} + \frac{f_H}{8} \cdot \frac{f_H}{f_0} - \frac{f_H}{128}\left(\frac{f_H}{f_0}\right)^3 + \dots\dots \qquad \dots \quad (8)$$

Hence approximately,

$$\Delta f = \frac{f_H}{2} + \frac{f_H}{8} \cdot \frac{f_H}{f_0} \qquad \dots \quad (9)$$

The critical frequency difference between the 0 and X components for the Q.T. mode of propagation is thus found to be dependent on f_0. If the value of f_H obtained from extrapolation of magnetic field at ground level is substituted in Eq. (9), Δf will be lower for higher values of f_0 and vice versa.

EXPERIMENTAL RESULTS

To find the gyro-frequency and its diurnal variations, if any, in the E, F1 and $F2$ regions, $h'-f$ records for the months of February to December 1955 (year of low sunspot activity) were examined. For the E and F1 regions, the records at intervals of 30 minutes were considered. Only the records in which simultaneous measurements of the ordinary and extraordinary critical frequencies could be measured accurately were considered. The critical frequencies (f_x, f_0) of E, F1 and, of the F regions at night, could be measured accurately upto $\pm$0.05Mc/s. This is because the logarithmic frequency scale in the $h'-f$ records permits higher accuracy at lower frequencies. Measurements for the higher values for the F2 region during day time were less accurate.

The average diurnal variations for the equinoxial months (March, April, Sept. and October) of f_H, as calculated from f_x and f_0 for the E, F1 and F2 region

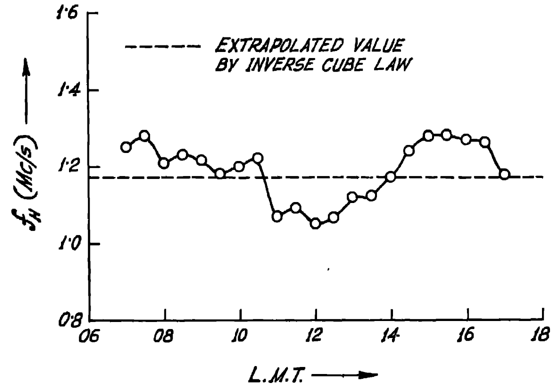

Fig. 3. Diurnal variation of the gyro-frequency (measured from f_x and f_0) in the E region

are shown in figures 3, 4 and 5 respectively. It is to be noted that in all the regions, the f_H values (average of all values) calculated from f_x and f_0 are higher than the

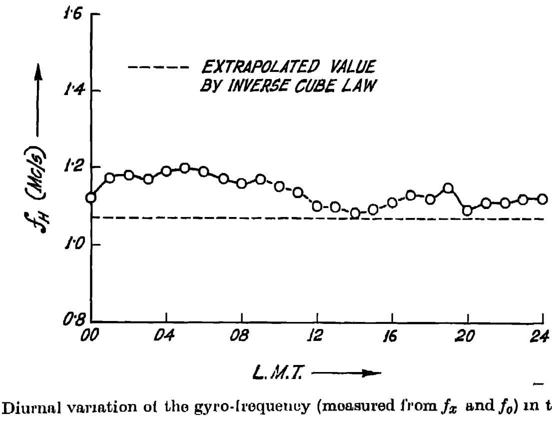

Fig. 4. Diurnal variation of the gyro-frequency (measured from f_x and f_0) in the F_1 region.

values expected by extrapolation of the magnetic field at ground-level. In the E region, f_H has a marked semi-diurnal variation with a midday dip while in the F1 and F2 regions no such regular semi-diurnal variation is found except that the midday values in the F1 region are lower.

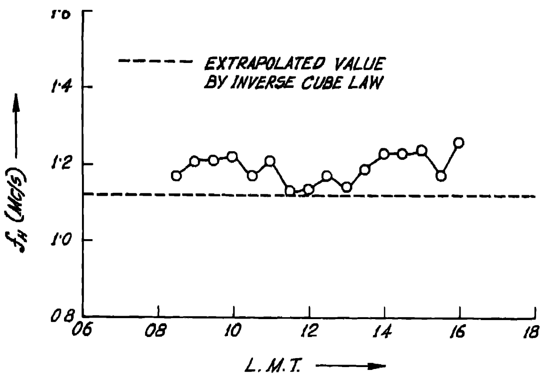

Fig. 5. Diurnal variation of the gyro-frequency (measured from f_x and f_0) in the F2 oregin.

Due to the high absorption of the extraordinary rays reflected from the F2 region during daytime, the number of records giving extraordinary critical frequencies during daytime was low. The diurnal variation curve of f_H for the F2 region between 0900 hr. and 1800 hr. (L.M.T.) is therefore shown by broken line.

Table 1 shows the average values (average of all values) of the gyro-frequency and the magnetic field calculated from f_x, f_0 for the equinoxial months in the E, F1 and F2 region and, the theoretical values of the same from the extrapolated magnetic field at ground-level by inverse cube law.

TABLE I

Region	f_H (from f_x, f_0) Mc/s	f_H (from inverse cube law) Mc/s	Magnetic field (from f_x, f_0) Gauss	Magnetic field (from inverse cube law) Gauss
E	1 20	1.17	0.43	0.42
F1	1.18	1.12	0.42	0.40
F2	1 14	1.07	0.41	0.38

Assuming the presence of heavy-ions in the E region, mean hourly values of the ratio (λ) have been calculated with the help of Eq. (5) from the measured values of f_x and f_0. For this purpose, the value of μ used was 4.33×10^4 which corresponds to an ion mass of 24. (This value of μ was also used by Scott in calculating the values of λ). Theoretical value of f_H used was 1.17 Mc/s (from inverse cube law). The mean hourly values of λ so calculated for the E region are shown in Table II.

TABLE II

Hour (L.M.T.)	f_0 Mc/s	f_x Mc/s	λ
07	2 30	3 00	−3830
08	2.70	3.37	−2120
09	3.13	3.80	−2560
10	3.20	3 85	−1090
11	3.52	4.09	6440
12	3.52	4.07	8760
13	3.60	4.20	2950
14	3.20	3.84	− 277
15	3.10	3.80	−4910
16	2.82	3.52	−4680
17	2.40	3.06	− 390

The calculated values of λ in Table II show that most of the hourly values (except near midday) of heavy-ion concentration are negative. This, in other words, means that assumption of the presence of heavy-ions leads to impossible results.

In the F region, the average nighttime value of the gyro-frequency is found to be 1.19 Mc/s in winter (Nov.—Dec.) and 1.09 Mc/s in summer (June- July). In this connection, it may be noted that when the f_H values in the F2 region for the equinoxial months are plotted with the height of maximum electron density, the plotted points do not show the expected variation (f_H decreasing with increasing height).

To find the dependence of Δf on f_0, only nighttime $h'-f$ records for the F region were examined when the F region critical frequency is low since these records permit greater accuracy of measurement. The values of Δf were found in three frequency ranges of f_0 lying between 1.5 Mc/s to 2.5 Mc/s, 3.5 Mc/s to 4.5 Mc/s and 5.5 Mc/s to 6.5 Mc/s with the centre f_0 values at 2.0 Mc/s, 4.0 Mc/s and 6.0 Mc/s respectively. The experimental results and the corresponding theoretical values expected according to Eq.(9) with the extrapolated value of f_H = 1.07 Mc/s at 300 km height are shown in Table III.

DISCUSSION

The gryo-frequency in the E region at Haringhata as obtained from the measurements of f_x and f_0 shows a marked semi-diurnal variation (figure 3) with a dip near midday. Similar semi-diurnal variations have also been found by Scott at Resolute Bay (Lat. 74.7°N) and Baker Lake (Lat. 64.3°N). The value of the magnetic field, corresponding to the dip in the gyro-frequency near midday at Haringhata is about 5000 gammas lower than the extrapolated value obtained from inverse cube law. The corresponding values as obtained by Scott are about 13000 and 7000 gammas at Resolute Bay and Baker Lake respectively. No satisfactory explanation of this diurnal variation of the deduced magnetic field has yet been offered. As shown before, calculation based on the assumption of heavy ions leads to impossible results. It may also be thought that as $(f_x - f_0)$ depends upon f_0 and as f_0 has a diurnal variation, the origin of the diurnal variation may be traced to the f_0 variation. But, the f_0 variation, even in extreme cases, amounts to only 55%, and this is insufficient to explain the magnitude of the variation.

TABLE III

Frequency range	$(f_x - f_0)$ (Experimental) Mc/s	$(f_x - f_0)$ (Theoretical) Mc/s
1.5 Mc/s to 2.5 Mc/s	0.63	0.61
3.5 Mc/s to 4.5 Mc/s	0.60	0.57
5.5 Mc/s to 6.5 Mc/s	0.55	0.56

One may also be tempted to ascribe the variation to the superposition of the magnetic field due to the wide-world upper atmospheric Sq current system on the magnetic field of the earth. Rocket measurements of the magnetic field at high altitudes [Maple *et al*, 1951; Singer *et al*, 1951 and Singer *et al*, 1952] have established the location of the current system near E region. If the current system is assumed to be an infinite horizontal plane current-sheet then the field due to it in the E region will be of the same order as that near ground, namely, about 30 gammas near midday. The observed diurnal variation of the gyro-frequency cannot, therefore, be an effect of the Sq. current system.

F region observations show a seasonal variation in the magnetic field. Thus, Baral and Mitra (1950) from observations of the critical frequencies over Calcutta had found the value of the magnetic field in Winter to be higher than in Summer. Scott has reported a large seasonal variation of 20%. Our observations show that Winter values are only 9% greater than the summer values. No explanation of this seasonal variation is yet available.

Scott has also obtained diurnal and other irregular variations of the apparent magnetic field (from f_x snd f_0) in the F region. Our observations, however, do not show such variations. The reason of this may be understood from the explanation of these variations as given by Scott. According to Scott the variations are due to the deflections of the ray paths from the vertical in conjunction with normal latitude gradients of ionization. He shows that the ordinary and extraordinary rays in the ionosphere follow curved paths, as a result of which they are reflected from areas which are displaced north and south of the transmitter. In the absence of electronic collision, the ordinary ray is deflected towards the pole while the extraordinary ray is deflected towards equator in the vertical plane of the earth's field. In the presence of collisions deflections of the ordinary and extraordinary rays have a small westward component. The horizontal separation of the ordinary and extraordinary reflection points affects the measured values of the critical frequency difference $f_x - f_0$. It is increased due to the separation of the reflection points for the usual variation of ionization with latitude (negative gradient of ionization density). As a result, the gyro-frequency, and hence the earth's magnetic field, calculated from $f_x - f_0$ is larger. Separation of the reflection points depends on the layer semi-thickness, the wave frequency, the magnetic field strength and the dip. It is negligible in the thin E and F1 region but is of considerable magnitude in the F2 region. Variation in the layer thickness (as is present in the F region) will produce a proportional change in the separation of the reflection points. Diurnal variations of the layer thickness, if any, will thus affect the values of f_H and of the magnetic field calculated from $f_x - f_0$. Scott (1950b) has calculated the separation of the reflection points for different values of the magnetic dip angles for a parabolic region. At Haringhata (Magnetic Dip $32°N$) the separation of the reflection points in the F region should be small. It is about 20 kms only at a wave frequency of 10 Mc/s (when electronic collisions are neglected) for normal values of F layer semi-thickness (≈ 50 km). The latitude gradient of ionization density is also small compared to that in the polar latitudes. Thus, the effect of ray-path deflections on the values of f_H and its diurnal variation in the F region over Haringhata calculated from $f_x - f_0$ is small.

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